THE MISSING LINK? – SCHOOL-RELATED CONTENT KNOWLEDGE OF PRE-SERVICE MATHEMATICS TEACHERS

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Professional knowledge is seen as a core component of teacher expertise. Hereby, domain-specific knowledge is usually modelled as content (CK) and pedagogical content knowledge (PCK). To date, the development of this knowledge during teacher education is so far not investigated comprehensively. The article focuses on a refined model of domain-specific teacher knowledge for that purpose that adds a school-related content knowledge (SRCK) as a specific applied mathematical knowledge for teaching. The article reports the development of an instrument to assess the professional knowledge. A study with N=505 pre-service teachers results in reliable and sufficiently separable scales for CK, SRCK, and PCK. SRCK seems to play an intermediary role between CK and PCK. The measures will be used to investigate the longitudinal knowledge development during teacher education. Practical implications are discussed.

INTRODUCTION AND THEORETICAL BACKGROUND

Professional knowledge of teachers is considered one core component of teacher expertise. From a domain-specific perspective, content knowledge (CK) and pedagogical content knowledge (PCK, Shulman, 1986; Baumert et al., 2010) are important aspects of this professional knowledge. Recent studies indicate that professional knowledge contributes to instructional quality and to student progress (Krauss et al., 2008; Kersting, 2010; Hill, Schilling, & Ball, 2005; Hill et al., 2008). Consequently, there is broad consensus that teachers' professional knowledge is a key goal of teacher education.

Nevertheless, the development of teacher expertise is still not comprehensively understood. Especially, there is a lack of research on the growth of teacher professional knowledge during initial teacher preparation. The project *KeiLa – Development of Professional Competence in University-based Teacher Education* aims to describe longitudinally the development of teacher knowledge from a broad perspective, including amongst others individual characteristics and learning opportunities across different domains (educational psychology, mathematics, biology, physics, chemistry). This interdisciplinary approach seems suited, as in several countries including Germany teachers major in two subjects and university-based teacher education includes education in educational psychology (cf. Lohse-Bossenz, Kunina-Habenicht, & Kunter, 2013).

One of the main challenges for research focusing on longitudinal effects of teacher education lies in the assessment of subject-specific knowledge. Although for mathematics, a few standardized tests of components of this knowledge were already

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developed, it can still be considered an emerging field, especially if a longitudinal perspective is taken. Existing approaches still differ widely, so that we conducted a study with preparing character (KiL – *Measuring the professional knowledge of preservice mathematics and science teachers*, Kleickmann et al., 2013) to develop instruments for the assessment of domain-specific professional knowledge. In this article, we focus on the mathematical part of the KiL-study. Therefore, we 1) review the state of research on (pre-service) teachers' content and pedagogical content knowledge (CK, PCK), 2) argue for the need of a complementing new construct of school-related content knowledge (SRCK), 3) report on the psychometric quality of the developed KiL-tests for pre-service teachers on CK, PCK, and SRCK and 4) present findings on the structure of professional knowledge as a whole and its components. Although the study is conducted in Germany, the focus on areas of domain-specific knowledge and its acquisition is seen as fundamental for mathematics teacher education in general.

The constructs of content knowledge and pedagogical content knowledge

Advancing the research of Shulman (1986), empirical studies were undertaken to operationalize the constructs of content knowledge (CK) and pedagogical content knowledge (PCK) for mathematics teachers. First investigations focused on the separability of the different domain-specific knowledge components as well as their importance for teaching quality and student learning. However, empirical studies could not completely answer the important questions concerning the structure of mathematics teachers' knowledge. In some studies for example CK and PCK are highly correlated (Hill et al., 2004, 2005; Krauss et al., 2008; Blömeke, Kaiser, & Lehmann, 2008). However, it is not always clear if this correlation is caused by the underlying conceptualizations, the different operationalisations or if it mirrors the nature of the investigated cognitive structures. For example, CK is often intended to mirror mathematics knowledge acquired through formal teacher education. Despite of this, most conceptualizations are predominantly focused on mathematical school content, even for teachers of academic track schools that receive a profound academic education in mathematics in most countries (Baumert et al., 2010; see also Tatto et al., 2012 for the structure of mathematics teacher education in 17 countries). In analogy, PCK is intended to mirror a kind of knowledge very specific for teaching mathematics. But operationalisations show that the delineation of PCK from analytical mathematical competences can be subtle (Buchholtz, Kaiser, & Blömeke, 2014).

Accordingly, one can ask if CK and PCK and the relation between these constructs are fully understood. Moreover, the existing approaches are not fully aligned with the aims of formal teacher educations. Thus, they are not suited to trace the effects of formal teacher education. Consequently, in the KiL study we furthered the conceptualizations of pre-service mathematics teachers' domain-specific knowledge to account for the depth and breadth of demands of mathematics teacher education.

In the KiL conceptualization, CK is conceptualized as academic mathematical knowledge, as expected to be acquired through formal teacher education. This mathematical knowledge is – in respect to content, precision, and notation – clearly beyond school mathematics. Students in mathematical study programs without aiming at a teaching license would also be expected to acquire this knowledge. Thus, this CK conceptualization refers to the original idea of Shulman (1986) who expected the "subject matter understanding of the teacher [to] be at least equal to that of his or her lay colleagues, the mere subject matter major" (p. 9). However, in line with modern teacher education programs, we would not expect a secondary teacher to complete a full mathematics major, but to have profound basic mathematical knowledge on the level of an introducing lecture in each major area of mathematics (e.g. analysis, algebra, geometry, applied mathematics) and further advanced knowledge in at least one major area with relevance for school mathematics. However, it is important to understand that our conceptualization of content knowledge is not restricted to elementary mathematics from a higher viewpoint (Klein, 1908).

Pedagogical content knowledge (PCK) refers to the knowledge about the instruction of specific mathematical topics. In KiL, we follow the suggestions of Baumert and colleagues (2010) and subsume knowledge of instructional strategies for a certain topic, knowledge about student cognitions, e.g. typical student misconceptions of a topic, and knowledge about the learning potential of specific mathematical tasks (Baumert et al., 2010). In other approaches, items were used to operationalize PCK that have a predominant mathematical demand (or could be solved by mathematical means, e.g. a mathematical argumentation; cf. Buchholtz, Kaiser, & Blömeke, 2013). But if PCK is understood as the knowledge "which goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge *for teaching*" (Shulman, 1986, p. 9, emphasis in original), we suggest to understand the conceptualization of PCK more rigorously. PCK then has to be a genuine and specific kind of knowledge about instruction, so it is per se knowledge about the teaching and/or learning of a certain topic and should clearly relate to student thinking. This, of course, has consequences for an operationalization of PCK, where mere mathematical problems should be avoided.

School-related content knowledge as a special kind of applied content knowledge

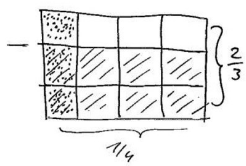
Using these conceptualizations of CK and PCK, we see the need for a complementing construct we call school-related content knowledge (SRCK). First, as neither CK nor PCK include knowledge about mathematical school contents and their curricular alignment, SRCK should encompass this knowledge. Curricular knowledge is commonly understood to be neither genuine PCK nor CK and some conceptualizations of teacher knowledge account for that knowledge explicitly (Shulman, 1987; Hill et al., 2005). But beyond, the sequencing of contents in specific curricula should inform instructional decisions. To solve such instructional problems, a cross-cutting subject-specific knowledge is needed: Answering questions of implications of curricular decisions needs specific knowledge about learning these topics as well as profound knowledge about the underlying connections that are caused by the deep mathematical structures, hence a knowledge intertwining content and pedagogical content knowledge. Here, two sub-facets can be identified: Teachers need to know how the topics of school mathematics are rooted in the mathematical structures and, vice versa, how mathematical structures can be reduced for teaching purposes (cf. "unpacking mathematics", Ball & Bass, 2003). As an example for the first facet, only the profound understanding of limits enables teachers to understand repeating decimals, especially the (non-trivial) validity of $0.\overline{9} = 1$. As an example for the other facet, the academic way of constructing real numbers via Cauchy sequences or Dedekind cuts is not suited for school mathematics. However, a profound mathematical knowledge helps connecting e.g. Cauchy sequences to the way irrational numbers are approximated with the help of nested intervals, a standard way to estimate the size of the square root of 2 at school. To sum up, we understand SRCK knowledge as a very special kind of application of mathematical knowledge for the teaching purpose. These ideas are informed by early reflections on the profession of mathematics teachers and the relation between academic mathematics and school contents (cf. meta-mathematics, e.g. Fletcher, 1975, Dörfler & McLone, 1986; cf. mathematical background theory, e.g. Vollrath, 1988).

Thus, we decided to conceptualize school-related content knowledge (SRCK) as a kind of applied mathematical knowledge for teaching that should be important to enable teachers to transform academic mathematical knowledge (CK) into knowledge for teaching mathematics at school and relate school mathematics to the structure of the discipline. It is questionable whether SRCK as an applied knowledge can be learned on its own. It seems that SRCK is deeply rooted in academic CK. At the moment, we do not see a well-defined place for the systematic development of this kind of knowledge in German teacher education programs. All the more, we see the need to investigate the development of this theoretically important knowledge area for pre-service teachers of mathematics.

INVESTIGATING DOMAIN-SPECIFIC PROFESSIONAL KNOWLEDGE OF PRE-SERVICE MATHEMATICS TEACHERS

In order to comprehensively assess pre-service mathematics teachers' domain-specific knowledge, we distinguish in our studies between the three dimensions of content knowledge (CK), school-related content knowledge (SRCK) and pedagogical content knowledge (PCK). We developed a test instrument building on this framework (see Figure 1 for sample items).

First, we conducted a curricular analysis of teacher education programs and curricula for school mathematics (both for secondary level, i.e. grades 5-13). Item development and piloting activities resulted in a total of 118 items (PCK: 31, SRCK: 34, CK: 54) that were bundled in two test booklets. One test booklet should be used with pre-service mathematics teachers for the academic track, the other for pre-service teachers for the non-academic track. However, both booklets had a considerable overlap of 81 items, in order to allow a linking of the data for analyses. The tests covered topics from arithmetics/algebra, analysis, geometry, stochastics, and numerics with a strong focus on arithmetics/algebra. With this, the test covers the characteristics of university-based teacher education as we could observe in the curricular analysis. Testing time was set to 120 minutes per booklet. The items were scored according to a scoring rubric with partly dichotomous, partly partial scores (0, 0.5, 1). For the 34 open answers, the interrater-reliability of the scoring of two independent raters was above $\kappa = 0.73$ (Cohen's Kappa), thus the objectivity of the scores was considered as sufficient.

<table>
<tr>
<td>PCK</td>
<td>Please sketch an iconic illustration to explain to a 6th grade student how to multiply a fraction by another fraction. Take $^1/_4 \cdot {^2/_3}$ as an example.</td>
<td>Sample Answer (scored as correct)


"Only the areas marked as  count. How many are these? How many are there in total? $^2/_{12} = {^1/_4} \cdot {^2/_3}$ "</td>
</tr>
<tr>
<td>SRCK</td>
<td colspan="2">Which of the following tasks enable students to discover that rational numbers are dense in real numbers?
<table>
<tr><td></td><td>apt</td><td>wrong</td></tr>
<tr><td>Measure the length of the diagonal of a square (length 10 cm).</td><td>□</td><td>×</td></tr>
<tr><td>Seek the smallest fraction bigger than $\sqrt{2}$.</td><td>×</td><td>□</td></tr>
<tr><td>Split 100€ into three equal portions.</td><td>□</td><td>×</td></tr>
<tr><td>Find ten fractions between $\sqrt{2}$ and $\sqrt{3}$.</td><td>□</td><td>×</td></tr>
</table></td>
</tr>
<tr>
<td>CK</td>
<td colspan="2">We call a field extension $L{:}\,K$ finite-dimensional if $dim_K L < \infty$. Which of the following field extensions are finite-dimensional?
<table>
<tr><td></td><td>right</td><td>wrong</td></tr>
<tr><td>$\mathbb{Q}(\pi){:}\,\mathbb{Q}$</td><td>□</td><td>×</td></tr>
<tr><td>$\mathbb{Q}(i){:}\,\mathbb{Q}$</td><td>×</td><td>□</td></tr>
<tr><td>$\mathbb{C}{:}\,\mathbb{R}$</td><td>×</td><td>□</td></tr>
<tr><td>$\mathbb{R}{:}\,\mathbb{Q}$</td><td>□</td><td>×</td></tr>
</table></td>
</tr>
</table>

Figure 1: Sample items for constructs of pre-service mathematics teachers' pedagogical content (PCK), school-related content knowledge (SRCK) and content knowledge (CK) tests

Sample and Methods

A total of N = 505 pre-service mathematics teachers participated in the study. On average, the students were 23.3 (SD = 2.9) years old and in their 5.9 semester (SD = 2.64). About 64% of the students aimed to teach in academic track schools (German Gymnasium). In order to investigate the structure of pre-service teachers' professional knowledge the dimensionality of the data was examined. Therefore, multidimensional random coefficients multinomial logit modelling was used (MRCML; Adams, Wilson & Wang, 1997). For the final analyses, 98 items could be maintained in the sense that they fulfil the required cutoffs for item quality indicators.

Results

The analyses presented here focus on the separability of the constructs CK, SRCK and PCK. Therefore, we contrast a three-dimensional model against a one-dimensional model (g-factor model). As the SRCK construct is seen as having a cross-cutting characteristics between CK and PCK, we further contrast two alternate two-dimensional models that combine SRCK with CK and PCK respectively (see Table 1). We could not apply chi-square test of differences to compare the fit of the different models, as they were not nested. Thus, we used the Bayesian information criterion (BIC). Smaller values indicate a better model fit. Raftery (1995, p. 141) counts a BIC difference greater than ten as “very strong evidence” and greater than six “as strong evidence” for the model with the lower BIC value.

The comparison of model fit indices indicates that the three-dimensional model fits the data best, outperforming the one-dimensional, and the two different two-dimensional models (see Table 1 for details). The three scales showed further satisfying EAP/PV reliabilities (r_{CK} = .83 with scale length 41, r_{SRCK} = .80 with scale length 31, r_{PCK} = .69 with scale length 26). Hence, we succeeded in measuring CK and PCK as well as a complementing SRCK component and the scales suggest sufficient reliability.

Model	Description	n	df	BIC
3D between model	CK – SRCK – PCK	112	44023.82	44720.97
2D between model A	CK/SRCK – PCK	109	44159.14	44837.62
2D between model B	CK – SRCK/PCK	109	44069.37	44747.85
1D general factor model	CK/SRCK/PCK	107	44312.97	44979.00

n = total number of estimated parameters, df = final deviance

Table 1: Comparison of alternate models

The latent correlation between PCK and CK was estimated as r(PCK,CK) = .54 indicating a good separability of the constructs. At the same time, SRCK correlated highly with both the CK (r(SRCK,CK) = .83) and the PCK (r(SRCK,PCK) = .85) dimension on the latent level. This can be seen as an indication that SRCK has indeed cross-cutting characteristics, as conceptualized.

DISCUSSION AND OUTLOOK

The results of the KiL study provided evidence for the postulated three-dimensional structure of pre-service mathematics teachers' domain-specific knowledge. On the basis of the refined constructs of CK and PCK, we were able to separate the two constructs satisfyingly on the empirical level. A complementing dimension of school-related content knowledge (SRCK) was conceptualized as a knowledge base for applying academic mathematical knowledge in the context of school mathematics and its instruction. On the empirical level, the correlations between the measures support this intermediary role of SRCK between academic mathematics and school mathematics. Thus, we were able to model pre-service mathematics teachers' domain-specific knowledge on the basis of the KiL model. With this we laid the groundwork to empirically investigate the growth of pre-service teachers' knowledge across formal teacher education using a longitudinal study in the upcoming KeiLa project.

On the basis of our findings, we would suggest to reinvestigate the value of academic mathematics for the development of teacher professional knowledge, a key element of teacher expertise. Our investigations might have importance for the design of teacher study programs. Especially, it is a new starting point to focus on an applied mathematical knowledge for teaching that is energized by a profound understanding of mathematics and enables a teacher to solve the evolving problems of teaching mathematics.

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