LEARNING FROM ERRORS: EFFECTS OF A TEACHER TRAINING ON STUDENTS' ATTITUDES TOWARDS AND THEIR INDIVIDUAL USE OF ERRORS

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A constructive handling with errors is considered as an important factor for learning processes. Accordingly, instruction should enable learners to develop positive attitudes towards error situations and to use errors as learning opportunities. In a quasi-experimental study with students from 31 classrooms (grades 6 to 9), we investigate effects on students' attitudes towards errors as learning opportunities in two conditions: (1) an error-tolerant classroom culture and (2) condition (1) with an additional teaching of strategies for analyzing errors. Our findings show positive effects of the error-tolerant classroom culture on the affective level, whereas students are not influenced by the cognitive support. Moreover, there is no evidence for differential effects for student groups with different attitudes towards errors.

INTRODUCTION

"Mistakes are often the best teachers.", "Aus Fehlern wird man klug.", "Erreur n'est pas crime." - In many languages all over the world, we find proverbs about errors. Interestingly, many of these proverbs attribute a positive function to errors. This indicates the existence of a cumulative human experience in which errors can have positive effects. However, many people associate negative feelings with errors, which probably arise from the fact that errors are one of the most important criteria to assess the performance of individual actions. Traditionally, mathematics education research has analyzed patterns underlying students' errors related to different mathematical concepts (e.g. Radatz, 1979; Tatsuoka, 1984). We want to stress that our perspective on errors differs from these specific diagnostic research perspective. Our goal is not to analyze why a learner makes an error and which individual misconceptions or problems are responsible for this. Instead, we focus on the error-handling activities that teachers and students perform in mathematics lessons. The main questions are how students experience the activities of their teachers in error situations, how students individually use their own errors as learning opportunities and which aspects of mathematics instruction are beneficial for motivating and supporting students' learning processes when dealing with individual errors in mathematics.

THEORETICAL BACKGROUND

The concept "error" can be defined as a process or a fact that does not match a given norm (Oser & Spychiger, 2005). As expressed by the proverbs quoted above, it seems

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to be a consensus in educational science that learning from errors is principally possible. One explanation for this assumption follows from the theory of negative knowledge because an understanding of errors is considered to be necessary for distinguishing between correct processes or facts and the incorrect surroundings.

Theory of Negative Knowledge and the Role of Errors

The theory of negative knowledge postulates that individuals accumulate two complementary types of knowledge: positive knowledge about correct facts and procedures and negative knowledge about incorrect facts and procedures (e.g., Minsky, 1994). Negative knowledge is necessary to identify the boundaries of correct facts and processes and therefore, to distinguish between correct and incorrect facts and processes. Since individuals are usually not taught about incorrect facts or processes, individual experiences in error situations are considered necessary to acquire this knowledge (cf. Oser & Spychiger, 2005). Nevertheless, it is questionable whether all error situations are fruitful learning opportunities and how the acquisition of negative knowledge for a future error prevention takes place.

Based on prior research (Garuti, Boero, & Chiappini, 1999; Heinze & Reiss, 2007; Oser & Spychiger, 2005), we propose a process model describing two different ways of dealing with errors (figure 1). We distinguish a pragmatic, outcome-oriented and an analytic, process-oriented path of action. While the former proceeds directly from error detection to error correction, the latter path includes a closer analysis of the error and the generation of error prevention strategies. With respect to generation of negative knowledge, the latter approach can be expected to be more beneficial.

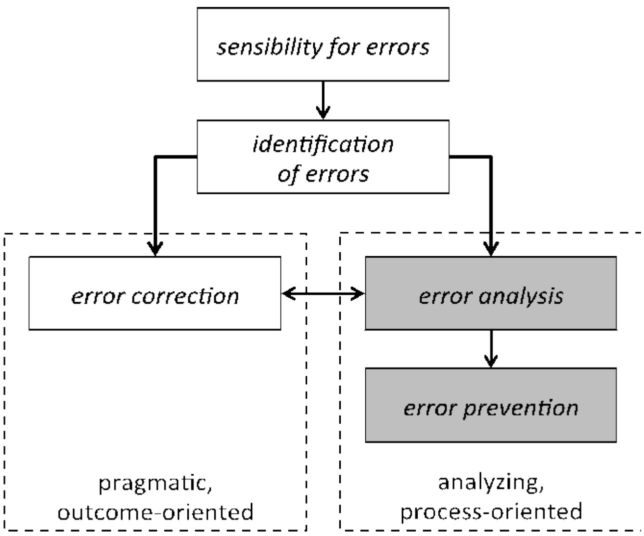

Figure 1: Process model for learning in error situations.

There is strong empirical evidence for this assumption from research on error management trainings in different domains of working life (Keith & Frese, 2008). Nevertheless, the choice of a productive, analyzing approach towards error situations does also rely on affective characteristics of the learner. Errors are often experienced as adverse events, and thus fear can impede their potentially positive effects. On the other hand, dealing with errors in learning showed to be more effective than avoiding them, if there is clear feedback and an error-tolerant culture (Keith & Frese, 2008; Nordstrom, Wendland, & Williams, 1998).

Error-handling Activities in the Mathematics Class

Findings from video-based investigations from Switzerland the USA, Italy and Germany (for a short survey see Heinze & Reiss, 2007) show similar tendencies how teacher deal with errors in mathematics class. Firstly, the number of errors in the public teacher-students interaction is comparatively low (about 3-5 errors per lesson). Secondly, about 90% of the public handling of errors in mathematics lessons are clearly teacher-directed. Oser and Spychiger (2005) explain the low number of errors per mathematics lesson with error avoidance behavior of teachers and students. In particular, teachers try to avoid interruptions in the ongoing instruction process and do not want to expose students who make errors. Accordingly, they pose their questions in such a way that students rarely give erroneous answers.

From the students' perspective on handling errors in mathematics class, there are hardly any empirical studies with a clear focus on this topic. Results from the Swiss group of Oser and colleagues based on questionnaire data from 295 4^{th}-9^{th} grade students indicate that students have a rather positive attitude towards dealing with errors during mathematics class as well as to the role the teachers play (Spychiger, Kuster, & Oser, 2006). The students perceive their teachers' error-handling as friendly and supportive and they rarely experience anxiety due to errors. However, they report only a moderate level for their individual use of errors as learning opportunities. Heinze, Ufer, Rach, and Reiss (2011) analyzed questionnaire data from 1674 students (grade 6-9). They showed that the perceived affective support of teachers correlates with lower anxiety and that the perceived cognitive support of teachers correlates with a more intensive use of errors as learning opportunities.

Heinze and Reiss (2007) investigate the effects of an in-service teacher training in 29 classes. Here, teachers of the experimental group received a combined training in error-handling and in teaching reasoning and proof, whereas the teachers of the control group only participate in a training on teaching reasoning and proof. The students were asked to evaluate how the teachers handled their errors. It turned out that only teachers of the experimental group classrooms improved their error-handling behavior significantly. Moreover, students' achievment in geometry proof increased significantly stronger in the experimental group than in the control group.

Borasi (1996) reports about several teaching experiments she conducted in a series of case studies. She developed a strategy of capitalizing on errors as springboards for inquiry in mathematics classrooms which is integrated in a specific teaching approach. In Borasi (1996) she summarizes that her case studies provide "anecdotal evidence" that learners can benefit from a specific teaching approach focusing the use of errors.

Summarizing these empirical studies, errors can be considered as an important factor of learning processes. However, students do often not use them as learning opportunities and it is unclear which aspects of mathematics instruction are relevant to change this. In particular, it is open if an error-tolerant classroom culture is sufficient to support students (affective support), or if in addition specific meta-cognitive strategies for a beneficial error-handling should be taught (cognitive support). Moreover, it is open if there are different groups of learners with specific perceptions of error situations so that differential effects of classroom interventions can be expected.

RESEARCH QUESTIONS

The aim of our research was to evaluate the effects of an error-tolerant classroom culture as well as specific interventions addressing strategies to use errors as learning opportunities. Moreover, we explored profiles of students' perceptions of error situations and the differential effects of the interventions on students with different profiles. Our study was guided by the following research questions:

- What are the effects of an error-tolerant classroom culture and interventions addressing strategies for learning from errors on students' attitudes towards error-handling?
- Is it possible to identify different types of students showing characteristic profiles with respect to their attitudes towards error-handling?
- If characteristic types of students can be identified, do the interventions affect students of different types in different ways?

DESIGN OF THE STUDY

We conducted a quasi-experimental intervention study with $6^{th} – 9^{th}$ grade students (12-15 years old) from 31 classrooms in different school types in Germany (comprehensive schools and schools from the academic focused school track Gymnasium). We applied a pre-post-questionnaire design with two experimental groups and a small control group. After cleaning the data from outliers, we obtained a sample of $N = 698$ students for the pre-questionnaire. Because of drop-out only $N = 571$ complete data sets were available for the pre- and post-questionnaire (cf. table 1).

Teachers of the *error-tolerant culture group* took part in a professional training program lead by the researchers. In this training, teachers were informed about the potential use of errors for learning and some of the empirical results described above. They were encouraged to consider errors as learning opportunities instead of useless interruptions in classroom. In particular, aspects of an error-tolerant and error-positive culture for mathematics classroom were discussed. Teachers of the *error-tolerant culture and strategies group* classrooms participated in the same training program and, in addition, they got materials dealing with strategies to learn from errors. These materials encourage learners to reflect on their errors made in homework or exercises and are based on the process model for learning from errors (right part in figure 1). In contrast to the first experimental group, these teachers got concrete ideas (and material) how they can support their students on a cognitive level. For an intervention check, teachers were asked to fill in a questionnaire how they used these materials. Teachers of the *control group* classrooms gave their regular lesson without any training and without using any additional material. Overall, the intervention took five months. The teachers in the second experimental group were asked to implement the material in their mathematics lessons regularly.

Intervention group	Classrooms	n_{pre}	$n_{pre\ \&\ post}$
Control group	4	87	73
Error-tolerant culture	13	267	218
Error-tolerant culture and strategy instruction	14	344	280

Table 1: Structure of the sample.

A slightly adapted version of the Swiss questionnaire on error handling in the mathematics classroom with 22 items (Spychiger et al., 2006) was used to assess students' attitudes towards error-handling before and after the intervention. As described in Heinze et al. (2011), four scales could be extracted (table 2).

Factor	Item example	Cronbach's α
Affective teacher support in error situations (TS_{aff})	*Sometimes our math teacher looks distressed when a student makes an error. [reversed item]*	.88/.91 7 items
Cognitive teacher support in error situations (TS_{cog})	*If I make an error in maths lessons my teacher handles the situation in a way that I can benefit from.*	.77/.83 4 items
No fear of making errors in mathematics lessons (NF)	*I become scared when I make an error in mathematics. [reversed item]*	.69/.73 3 items
Individual use of errors for the learning process (IU)	*In mathematics I explore my errors and try to understand them.*	.78/.81 8 items
Likert scale: 0 = strongly disagree, 1 = disagree, 2 = agree, 3 = strongly agree		

Table 2: Student questionnaire: Item examples and reliabilities (pre/post).

RESULTS OF THE STUDY

To evaluate the effects of the interventions, we conducted ANCOVAs for each of the four scales with the pre-questionnaire results as covariate and the group as a factor. There was a general regression towards disagreeing ratings from pre- to post-test, that has to be taken into account when interpreting the questionnaire data. There was no effect of the intervention on individual use of errors (IU, $F(567,2) = 1.338$, $p > .05$) and on cognitive teacher support (TS_{cog}, $F(567,2) = 0.495$, $p > .05$). The intervention had significant positive effects on affective teacher support (TS_{aff}, $F(567,2) = 9.476$, $p < .001$, $\eta^2 = .032$) and reduced the fear of making errors (NF, $F(567,2) = 3.765$, $p < .05$, $\eta^2 = .013$). There are no significant differences between the two experimental groups in these variables, but students from the experimental groups reported less fear and more affective support than the control group students (table 3).

M (SD)	Control group		Error-tolerant culture		Error-tolerant culture and strategy instruction	
	Pre	Post	Pre	Post	Pre	Post
TS_{aff}	1.71 (0.84)	1.47 (0.96)	2.03 (0.77)	2.02 (0.83)	2.12 (0.69)	2.12 (0.77)
TS_{cog}	1.91 (0.62)	1.74 (0.73)	1.84 (0.77)	1.79 (0.83)	1.91 (0.68)	1.81 (0.78)
NF	1.98 (0.77)	1.93 (0.87)	2.01 (0.73)	2.18 (0.71)	2.12 (0.74)	2.21 (0.75)
IU	1.78 (0.45)	1.57 (0.56)	1.71 (0.49)	1.64 (0.48)	1.68 (0.55)	1.60 (0.57)

Likert scale: 0 = strongly disagree, 1 = disagree, 2 = agree, 3 = strongly agree

Table 3: Development of students' perceptions of error situation.

To describe students' profiles, we carried out a cluster analysis using the four scales of the pre-questionnaire (Ward method). We could identify three types of learners with respect to their attitudes towards error-handling (figure 2), two types showing relatively high resp. low ratings on all scales and one type reporting some teacher support, little fear of errors, but also little use of errors as learning opportunities.

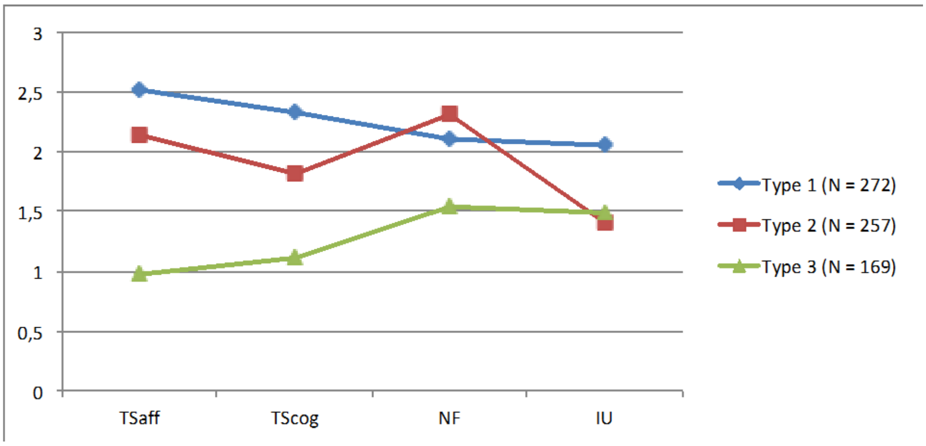

Figure 2: Profiles of the student types identified in the pre-questionnaire.

A MANCOVA with the four scales of the post-questionnaire as dependent variables, the corresponding scales in the pre-questionnaire as covariates, the intervention group and the student types as factors shows a significant effect of the intervention group ($F(1112,8) = 5.059$, $p < .001$, $\eta^2 = .035$) but no effect of student type

($F(1112,8) = 0.674$, $p > .05$) and no interaction effect between the two ($F(2232,16) = 1.220$, $p > .05$).

DISCUSSION

For teaching and learning in school and, in particular, in the mathematics classroom, errors are often considered as an important part of the learning process. In the presented study, teachers of the experimental group classes took part in a training about the role of errors and the importance of an error-tolerant classroom culture. In addition, some of these teachers implemented learning material in their lessons that encourage students to analyze their errors so that they can develop error prevention strategies (right part of figure 1). So in both intervention groups students got an affective support and in the second intervention group an additional cognitive support was provided.

Concerning research question 1, our findings show that - in comparison to a control group - there is a positive effect of both interventions with respect to students' fear of making mistakes and students' perception of affective teacher support. This indicates that the teachers of the experimental groups were able to implement an error-tolerant classroom culture and that this change had positive effects for their students on an affective level. However, a comparison of the two intervention groups does not give evidence for the influence of the additional systematic cognitive support. In particular, students' perception of teachers' cognitive support and students' reports on their individual use of errors do not change. We observe the same result when comparing the intervention groups with the control group. It seems that an affective support based on an error-tolerant classroom culture is not sufficient for inducing a change of students' perception of and handling in error situations with respect to the cognitive level.

The surpising result is that students of the second intervention group got a cognitive support but they did not notice this. One reason might be the unfamiliar demands for students when working with the implemeted material on analyzing errors. In Germany, students are not familiar with reflections about their own learning processes and, in particular, with reflections about their own errors. A possible second reason can be the quality of the intervention. Since we have only questionnaire-based self reports of the teachers, we do not know if they used the learning material in an adequate way.

Concerning research questions 2 and 3, we were able to identify three types of learners which report different perceptions of and handling in error situations during mathematics learning. However, the results of the MANCOVA did not reveal differential effects of the intervention on these three types of learners. Accordingly, for all three types we observed similar positive effects on the affective level and no effects on the cognitive level. Based on these findings, we assume that there is no need for a student type-specific affective support by an error-tolerant classroom culture. Concerning the cognitive support in error situations, the investigation of student type-specific instruction strategies might be a promising idea for further research.

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