TEXTBOOK EFFECTS ON THE DEVELOPMENT OF ADAPTIVE EXPERTISE

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During the last two decades research on the development of adaptive expertise has gained growing research interest. While a number of studies investigated the effects of different instructional approaches, the state of knowledge regarding the impact of learning resources in this field is quite limited. This study provides new insights into the relations of textbook quality and students' adaptive use of strategies in multi-digit addition and subtraction. By reanalysing longitudinal data of 1404 students from grade 1–3, we found quality discrepancies in the textbooks' opportunities to learn as well as substantial effects of these on the students' actual strategy use. Thus, mathematics textbooks can be regarded as meaningful classroom factor predicting the development of students' adaptive expertise.

THEORETICAL BACKGROUND AND EMPIRICAL FINDINGS

In the last two decades, a vigorous research interest in the genesis of individual adaptive expertise developed in mathematics education and psychology. Although valuable progress has been made, especially by the implementation of reform-based instructional approaches, little is known yet about the role of learning resources in this domain. According to Mullis and colleagues (2012), textbooks can be considered as the most important teaching materials used by primary school mathematics teachers. However, relational or even causal research on textbooks is limited (Fan, 2013), and the findings about effects on students' achievements are inconsistent (van Steenbrugge et al., 2013; Törnroos, 2005).

Adaptive expertise in multi-digit addition and subtraction

Since end of the 1990's, there has been a development towards a broad consensus about the importance of an adaptive or flexible use of strategies in arithmetic computation as a learning goal in primary school mathematics. A comprehensive definition of adaptivity is given by Selter (2009, p. 624):

> Adaptivity is the ability to creatively develop or to flexibly select and use an appropriate solution strategy in a (un)conscious way on a given mathematical item or problem, for a given individual, in a given sociocultural context.

In the context of multi-digit addition and subtraction up to 1000 we distinguish between mental calculation, written algorithms and informal strategies (e.g., Heinze et al., 2009). Here, informal strategies cannot unambiguously be assigned to mental or written methods due to a fluent transition (Fuson et al., 1997). An overview of the most common solution strategies is given in Table 1 (Selter, 2001). Adaptive expertise in

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this context is indicated by an adaptive use of strategies to find efficient solutions to given arithmetic problems.

Stepwise	Split	Compensation	Simplifying	Indirect Addition
123 + 456 = 579	123 + 456 = 579	527 + 398 = 925	527 + 398 = 925	701-698 = 3
123 + 400 = 523	100 + 400 = 500	527 + 400 = 927	525 + 400 = 925	698+3 = 701
523 + 50 = 573	20 + 50 = 70	927 – 2 = 925		
573 + 6 = 579	3 + 6 = 9			

Table 1: Most common types of strategies for addition and subtraction (the table shows examples for addition; there are corresponding versions for subtraction for all strategies but indirect addition).

First longitudinal studies revealed that adaptive expertise is accompanied with a broader conceptual knowledge or a deeper understanding of base-ten number conceptions (e.g., Fuson et al., 1997). In a row of reform-based teaching experiments, the development of students' strategy use was examined, yielding first insights, like positive effects of an early emphasis on the flexible use of strategies in lessons and of comparing and contrasting the efficiency of different strategies (e.g., Klein, Beishuizen & Treffers, 1998; Rittle-Johnson & Star, 2007). A theoretical framework for the genesis of adaptive expertise was developed by Siegler and his associates (cf. Siegler & Lemaire, 1997). The framework distinguishes four dimensions, for which are shown that changes in any of them can improve speed and accuracy of strategy choice overall. The four dimensions are: *strategy repertoire*, the knowledge of different types of strategies; *strategy distribution*, the knowledge of relative frequencies these strategies are used; *strategy efficiency*, the ability to perform strategies quickly and accurately; and *strategy selection*, the ability to flexibly select a strategy on a given problem.

Nevertheless, empirical studies repeatedly reported a lack of adaptivity in students' actual strategy use (e.g., Selter, 2001; Heinze et al., 2009; Torbeyns & Verschaffel, 2016). Although a shift towards the weight of informal strategies is perceptible in mathematics education, like the adoption in curricula and standards, students still tend to favour one strategy per operation, which is applied to almost each type of problem. One reason for the missing success might be a lack of learning opportunities or at least a lack of quality of learning opportunities in the mathematics classroom.

Textbooks as learning resources

For the evolution of adaptive expertise, learning resources like textbooks could play an important role. As mediator between the official and the implemented curriculum, textbooks translate the abstract curriculum into concrete operations for teachers and students to carry out (Valverde et al., 2001). Thus, as the adaptive use of strategies was

adopted in many curricula and standards, textbooks should offer opportunities to learn for this domain. In turn, the textbook content influences the teachers' instruction, as they're the most important learning resource for primary school mathematics teachers (Mullis et al., 2012). In the Trends in International Mathematics and Science Study (TIMSS) 2011 86 % of the German and 75 % of all primary school mathematics teachers declare to use the textbook as a basis of instruction (Mullis et al., 2012). Moreover Krammer (1985) found differences in the implemented teaching practices of teachers using distinct textbooks. Schmidt et al. (2001) found a relationship between the space a topic covers in a textbook and the instructional time teachers dedicate to this topic in the mathematics classroom. Those topics which are not included in the textbooks used are unlikely to appear in classroom (Schmidt et al. 1997). Furthermore, there are some indications that textbooks have an effect on student achievement. Schmidt et al. (2001) found a direct relation between the amount of space allocated to covering a topic and the size of achievement gain on that topic for the eighth grade TIMSS data of the United States. In line with that, Törnroos (2005) showed an effect of the number of opportunities to learn in mathematics textbooks for the TIMSS test on the students' achievement in TIMSS. Using the sample of our study presented below, Niedermeyer et al. (2016) found substantial differences of four different textbooks on students' arithmetic achievement from the end of the first grade till the end of the second grade. On the other hand van Steenbrugge et al. (2013) did not find any differences in the performances of elementary school students using distinct mathematics textbooks. All in all, there are some indications that characteristics of mathematics textbooks affect the student achievement. In the special field of adaptive expertise, some studies suggest such an effect on the students' strategy choice (cf. Fagginger Auer et al., 2016; Heinze et al., 2009). While Fagginger Auer and colleagues showed the relation of the textbook used and the student's strategy profile in a multilevel latent class analysis, Heinze et al. found a relation between the adaptivity of student's strategy use and their textbook's instructional approach to teach adaptive use of strategies.

According to Fan (2013), there is still a lack of evidence-based relational and causal research on the effect of mathematics textbooks. Although the studies reported may give valuable indications, their scope remains cross-sectional (except for Niedermeyer et al., 2016). However, for the investigation of a prolonged process like the development of adaptive expertise, there's a need for longitudinal data. Also, previous research on textbooks is mainly small-scale and often includes textbooks representing different curricula, with the result that the effects of curricula and textbooks are confounded. Furthermore, to our knowledge, there are neither any qualitative textbook analyzes developing a scale to classify textbook quality, nor any studies examining the effects of textbook quality on student achievement (Fan, 2013). Most existing studies of textbooks' contents refer to comparisons between books, often between those of different countries (Fan, 2013).

RESEARCH QUESTIONS AND METHODS

The present study aims at contributing to the mentioned research gaps. In contrast to most former investigations on textbooks effects, we use longitudinal data (grade 1-3) of a large sample. By comparing textbook series following the same curriculum, we can circumvent the problem that curricula and textbooks are confounded. Another research desideratum we address is the development of a textbook quality scale and the examination of the effects of textbook quality on student achievement. Since a global measure for textbook quality seems barely feasible and adaptive expertise is an important learning goal, this study focuses on the subdomain of adaptive expertise. Consequently, we address the following research questions: (1) Do textbook series differ with respect to the quality of their opportunities to learn for adaptive expertise? (2) Which effect does the textbooks' quality concerning adaptive expertise have on students' adaptive expertise at the end of grade 3?

Research context

The basis for our analysis is an existing data set from a large three-year longitudinal study with primary school students from one federal state in Northern Germany. The overall sample consists of 2330 students from 127 classes. It comprises student data from the beginning of grade 1 (at the age of 6 years) to the end of grade 3. The original aim of the study is to address students' development in arithmetic. Of this sample, about 1700 students from 82 classes use one of the four most common mathematics textbooks: "Denken und Rechnen", "Einstern", "Flex und Flo" and "Welt der Zahl". The distribution of the classes over the textbooks is relatively even. Our analysis is based on the subsample of 1404 students from the 82 classes, who use one of the four textbooks and worked on the tasks examining adaptive expertise at the end of grade 3.

Instruments, data collection and analysis

For the purpose of a quality-based scale of the four textbook series we derived categories from the previously mentioned dimensions of adaptive expertise by Siegler and Lemaire (1997). Since strategies for multi-digit addition and subtraction are taught in grade 2 (number domain up to 100) and grade 3 (number domain up to 1000), we analyzed both books of each series by three independent and trained persons on the basis of the categories derived. A uniform scoring was reached by consensus method. In each category, we set up a ranking and compiled a relative overall scale of the books' quality for both grades by weighting each category equally. The following four categories were derived and scored: *(1) Strategy repertoire*: 0 points if a strategy was not treated, 1 point for incidentally introduced strategies (e.g., as a "trick"), 2 points for explicitly introduced strategies, and 3 points if explicitly introduced strategies were additionally illustrated in different representations; (2) *Strategy distribution*: 0 points for strategies not introduced or introduced only once (as this is included in the preceding category), 1 point for strategies introduced and additionally presented by means of another task or problem, and 2 points for strategies introduced and additionally presented more than once by means of other tasks or problems; (3)

Strategy efficiency: Since strategy efficiency evolves and enhances by growing experience and practice, our analysis criterion would have been practicing problems and task, to foster this ability. As we didn't find any variance between the textbooks regarding their amount of exercises, we did not include strategy efficiency as a category to our analysis; (4) *Strategy selection*: Former research has shown that an explicit comparison of solution strategies is conducive for an adaptive use of strategies (cf. Rittle-Johnson & Star, 2007). Therefore, we assessed dichotomously whether exercises explicitly demand a strategy comparison as a systematic component (i.e., a recurring content) of the books, or not.

Student and teacher data were collected by different tests and questionnaires. Data for controlling the learning prerequisites of the students (basic numerical skills, basic language skills, general cognitive abilities) were measured with approved standardized instruments at the beginning of grade 1. Data for the individual learning progress were collected at the end of grade 1 and 3 with grade-specific arithmetic tests. The arithmetic test at the end of grade 1 was scaled using Item Response Theory. The arithmetic test at the end of grade 3 included four problems (482+218, 473+398, 381-99, 702-698) each suggesting the use of specific strategies as efficient solutions (e.g., indirect addition for 702-698). For each problem, the written calculations or notes of the students were coded by trained research assistants with partial credit scoring for inefficient (0 points), partly efficient (1 point), or efficient (2 points) strategy use. Purely mentally computed solutions without notes were considered as the use of an (internalized) efficient strategy. About 22 % of the solutions with notes were double-coded, providing solid Cohen's κ's of .83–.90. The resulting scale for students' adaptive expertise (0–8 points) shows an acceptable reliability (Cronbach's $\alpha = .71$).

We conducted multilevel analyses which take into account the nested structure of the sample (students in classes). We included the variables for learning prerequisites at school entrance on the individual level and cognitive ability also as aggregated value on the class level (as an indicator of group composition). To account for the arithmetic development we included the grade 1 arithmetic test scores on individual level. Teachers beliefs (whether they're rather constructivist or not), a scale combining teacher experience and qualification, as well as the previously described textbooks quality for grade 2 and 3 were included on class level. Missing data on independent variables were handled by the Full Information Maximum Likelihood method (FIML). Due to sample selection we had no missing data for the adaptive expertise score.

RESULTS

The analysis of textbook quality yielded scores from 10 to 24 points per book regarding the dimension *repertoire*, 4 to 10 points per book regarding *distribution* and 0 to 1 point per book regarding *comparison*. While the books of the series "Welt der Zahl" are ranked first in all categories but one, those of "Einstern" are ranked last with only one exception. The relative overall scales of the books' quality were derived by an equally weighting and averaging of the three category rankings. The final mean ranks

range from 1.00 ("Welt der Zahl") to 2.33 ("Einstern") in grade 2 and from 1.33 ("Welt der Zahl", "Flex und Flo") to 3.33 ("Einstern") in grade 3. In relation to each other, the opportunities to learn for adaptive expertise of the "Welt der Zahl" series are of the highest quality on this scale, those of "Einstern" of the lowest, the other two are in between. With respect to research question 1 we have found substantial differences in the textbooks' qualities.

The outcomes of the multilevel analysis with students' adaptive expertise as dependent variable are shown in Table 2. In model 1 a substantial effect of arithmetic prior knowledge on individual and a small significant effect of the class composition regarding basic cognitive abilities on class level can be seen. The model 2 also includes the teacher variables which have no significant effects. The largest significant effect appears by including the textbook quality scales of grade 2 and 3 in model 3. The effect of the textbook quality in grade 2 is substantial, whereas the textbook quality in grade 3 has no additional effect. The inclusion of the textbook quality led to a substantial increase of the explained variance from model 2 to model 3 ($\Delta R^2 = 11.4$).

	Model 1	Model 2	Model 3
Level 1 (students)			
Basic cognitive abilities	.01 (.01)	.01 (.01)	.01 (.01)
Linguistic preconditions	-.01 (.02)	-.01 (.02)	.01 (.01)
Mathematical preconditions	-.02 (.01)	-.02 (.01)	-.02 (.01)
Arithmetic prior knowledge	.48** (.06)	.48** (.06)	.48** (.06)
Level 2 (class)			
Basic cog. abilities (aggregated)	.14** (.04)	.13** (.04)	.12** (.04)
Arithmetic prior knowl. (aggr.)	-.30 (.18)	-.25 (.18)	-.30 (.19)
Teacher qualification		-.32 (.17)	-.25 (.15)
Teacher beliefs		.04 (.26)	-.01 (.20)
Textbook quality (grade 2)			-.50** (.19)
Textbook quality (grade 3)			-.09 (.16)
Intercept	-1.60 (1.11)	-1.10 (1.24)	-1.89 (1.23)
Explained within class variance	11.1 %	11.1 %	11.1 %
Explained between class variance	24.4 %	31.2 %	42.6 %

* p <.05, ** p <.01,

Table 2: Multilevel regression for individual and classroom covariates and textbook quality on students' adaptive expertise at the end of grade 3

DISCUSSION

While the results presented are plausible according to the assumption that arithmetic prior knowledge and the quality of opportunities to learn are predictors for students' adaptive expertise, they provide new insights regarding the role of textbooks as learning resources in this field. We have not only shown effects of the textbooks on

students' adaptive expertise, but also, that these effects can be explained by the quality of the textbooks' opportunities to learn. The results reveal that a fine-grained and theory-based content analysis of mathematics textbooks is a fruitful approach to understand the impact and effects of textbooks as learning resources. Our findings supplement the results of Schmidt et al. (2001) and Törnroos (2005). Following this approach the outcomes may be replicated for other mathematics domains.

Our results have important implications. The present study shows that students using a certain textbook could be disadvantaged in comparison to students using another textbook. Therefore a textbook permission, based on theory based quality indicators, could lead to an improvement of textbook quality and avoid a disadvantage of students caused by a specific textbook. Furthermore, teachers should be trained how to use textbooks, so they are able to reflect on the quality of textbooks' learning opportunities and, if necessary, compensate inadequate representations of the curriculum.

There are several limitations of our study. Since we reanalysed an existing data set we were not able to administer specific instruments for our research. In particular, the questionnaires do not provide fine-grained data on the implementation of the teaching content or the teacher knowledge. Despite of these limitations the data set has the advantage that it covers a large sample taught by the same curriculum and allows multilevel analysis with an adequate explanatory power. Furthermore we were able to assess and examine the effects of textbook quality on student achievement for a specific domain. Accordingly, we were able to supplement and further develop existing research on the effects of textbooks on students' learning.

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